THREE TYPES OF TASKS
- EXTENDED INVESTIGATIONS
- IN-CLASS INVESTIGATIONS
- INVESTIGATIVE QUESTIONS

WRITTEN BY TEACHERS FOR TEACHERS

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18 ASSESSMENT TASKS

INCLUDES SOLUTIONS WITH MARKING KEYS SHOWING MATHEMATICAL BEHAVIOURS

Australian CURRICULUM

The Mathematical Association of Western Australia Inc.

FREE SAMPLE TASK

INVESTIGATIONS FOR TEACHING & LEARNING YEAR 12 GENERAL MATHEMATICS

Foreword

For each unit three types of tasks are included:

- **Extended teaching and learning investigation** – an investigation that includes a preparation activity which could be done in class, individually or in groups, in the student's own time **followed by** an in-class validation, so that students can reflect on their understanding
- Solutions are provided where practicable for the preparation activity/investigation. For the in-class validation, there are solutions and marking keys, which identify the mathematical behaviours that students may exhibit.
- Teachers are encouraged to share these with students, so they gain experience with WACE style marking keys.

- **In-class teaching and learning investigation** – an investigation for which no prior preparation is required.
- Solutions and marking keys are provided.
- This type of task will give students experience with investigative style questions that may be included as part of response type assessments.

- **Investigative questions** – a series of short questions, which test the student's ability to apply their learning, to justify their conclusions, to investigate and to generalise, or to solve problems.
- Such questions could be included in a response or examination assessment. Solutions and marking key are provided.

Screen shots have been produced using the CASIO ClassPad II emulator

Most graphs and diagrams have been produced using efofex software.

Contents

MATHEMATICS APPLICATIONS: Year 12

TASK 15

Investigative questions

Unit 4

Topic 2: Loans, investments and annuities

Course-related information

The concepts and skills covered in this investigation relate to the following content descriptors within the Australian Curriculum: General Mathematics course:

use a recurrence relation to model a compound interest loan or investment and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment (ACMGM094)

calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)

use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan (ACMGM097)

The ability to choose and use appropriate technology to enhance and extend concept development is also required for some of the items

Background information

The ability to create and interpret spreadsheets is assumed knowledge for the first questions. Students should be familiar with the concepts of the loans attracting interest calculated using a compound interest model, repayments, frequency of compounding and the calculation of effective interest rates.

Task conditions

These items should be undertaken under test conditions, either as part of a test/examination or as stand-alone items. Student access to a graphical/CAS calculator is assumed. The time required to complete each question is left to the discretion of the teacher but the intention is that each question may be completed within 15 minutes.

Investigative questions for Topic 2

Question 1 **(10 marks)**

The table below shows part of a spreadsheet containing data about a loan including the repayments and the amounts owed. The interest rate is the same for both loans.

The data are as follows:
Column A: Time elapsed (months)
Rows 3-12 of column B: Amount owing on first loan at the end of each month
Rows 3-12 of column C: Amount owing on second loan at the end of each month
(Amounts are given to the nearest dollar.)

	A	B	C
1	**Month**	**500**	**100**
2	0	**10000**	**10000**
3	**1**	$9,542	$9,942
4	**2**	$9,081	$9,883
5	**3**	$8,619	$9,824
6	**4**	$8,155	$9,765
7	**5**	$7,689	$9,706
8	**6**	$7,221	$9,646
9	**7**	$6,751	$9,587
10	**8**	$6,279	$9,526
11	**9**	$5,806	$9,466
12	**10**	$5,330	$9,406

(a) How much was owed on the first loan after nine months? (1)

(b) After how many months had nearly $300 been paid off the second loan? (1)

(c) Describe what the data in cell C11 represents. (2)

(d) The formula for calculating the contents of cell B5 is
=B4*(1+0.05/12) - B1
where * is the symbol for multiplication and / is the symbol for division.
Show the substitution needed to calculate the value in B5 (2)

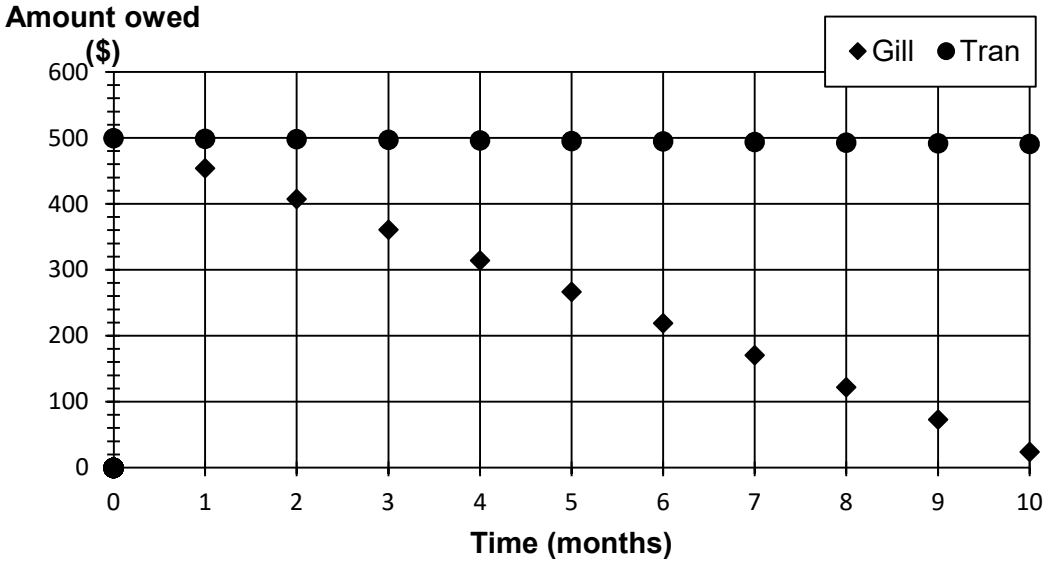
(e) What does the value in B1 represent? (1)

(f) Why do the amounts in column B decrease more quickly than those in column C? (1)

(g) Using the symbols defined in part (d) if needed, determine the formula to calculate the contents of C5. (2)

Question 2 **(11 marks)**

The graph below shows the amounts owed by Gill and Tran at the end of each month for the first ten months of their loan period. The interest rates were the same (10% per annum) for both loans and both were calculated using a compound interest model.



(a) Give an estimate for the amount owed by Gill after four months. (1)

(b) Estimate the difference in what the two people owe after two months. (1)

(c) Gill is paying back $50 per month on a $500 loan. Explain why the loan has not been paid off after 10 months. (1)

(d) Determine the formula to calculate the amount owed by Gill after n months. (3)

(e) Compare the two loans by summarising their similarities and differences. (4)

(f) Explain the differences in the amounts owed by these two people over the 10-month period. (1)

Question 3 **(9 marks)**

The table below shows the values of an investment of $50,000 at 6.00% per annum after six months, one year and two years for different compounding periods. The compounding periods are daily, weekly, monthly and yearly. Values have been rounded to the nearest dollar.

		Daily	**Weekly**	**Monthly**	**Yearly**
$50,000.00					
6.00%	Interest	0.000164384	0.001153846	0.005	0.06
	After				
	6 months	$51,527	$51,522	$51,519	
	1 year	$53,092	$53,090	$53,084	$53,000
	2 years	$56,374	$56,371	$56,358	$56,180
Effective annual interest rate		6.1831%	6.1800%	6.1678%	**A**

(a) What was the value of the investment after two years when interest was compounded monthly? (1)

(b) By how much was the investment increased after one year when the interest was calculated daily rather than weekly? (1)

(c) Use evidence from the table to support the statement below.

The value of this investment was increased by compounding more often. (2)

(d) How was the figure of 0.000164384 calculated? (1)

(e) What value should be written in the cell marked **A**? (1)

(f) Summarise the effect of increasing the frequency of compounding on the effective interest rate. (1)

(g) Estimate a value for the effective annual interest rate when compounding occurs fortnightly. Justify your estimate using data from the table. (2)

Investigative questions in Topic 2
Solutions and marking key

Question 1(a)

Solution	
$5806	
Mathematical behaviours	Marks
• reads table of values	1

Question 1(b)

Solution	
7	
Mathematical behaviours	Marks
• reads table of values	1

Question 1(c)

Solution	
Amount owing on the second loan after 9 months	
Mathematical behaviours	Marks
• identifies the amount and the type of loan	1
• identifies correct time period	1

Question 1(d)

Solution	
=9081*(1+0.05/12) - 500	
Mathematical behaviours	Marks
• substitutes 9081 into formula provided	1
• substitutes 500	1

Question 1(e)

Solution	
Monthly repayment	
Mathematical behaviours	Marks
• identifies contribution to loan	1

Question 1(f)

Solution	
Repayments are higher for the first loan	
Mathematical behaviours	Marks
• identifies higher repayments	1

Question 1(g)

Solution	
=C4*(1+0.05/12)-C1	
Mathematical behaviours	Marks
• uses correct formula	1
• substitutes cell references	1

Question 2(a)

Solution	
$310	
Mathematical behaviours	Marks
• reads value from graph	1

Question 2(b)

Solution	
$100	
Mathematical behaviours	Marks
• reads values from graph	1

Question 2(c)

Solution	
Interest is being added to the loan	
Mathematical behaviours	Marks
• identifies the addition of money to the loan	1

Question 2(d)

Solution	
Amount owed = $500 (1 – 0.1 ÷ 12)^n$	
Mathematical behaviours	Marks
• uses compound interest (or recursive) formula	1
• identifies starting amount of 500	1
• determines monthly interest rate	1

Question 2(e)

Solution	
The loans are similar in that $500 is borrowed in each case, the annual interest rate is the same (10%), the compounding period is the same (monthly), the loan repayments must be different.	
Mathematical behaviours	Marks
• identifies three similarities	3
• identifies difference	1

Question 2(f)

Solution	
Repayments are different – Tran is paying back the loan more slowly	
Mathematical behaviours	Marks
• identifies different repayments	1

Question 3(a)

Solution	
$56,358	
Mathematical behaviours	Marks
• reads table of values	1

Question 3(b)

Solution	
$92	
Mathematical behaviours	Marks
• determines difference in values on table	1

Question 3(c)

Solution	
After 6 months the values went from $51,519 up to $51,522 up to $51,527. After one year the increases went from 3000 to $3,084 to $3,090 to $53,092 There was a similar increase after 2 years.	
Mathematical behaviours	Marks
• identifies increase for each compounding period • identifies the increase occurs after each investment period	1 1

Question 3(d)

Solution	
=0.06 ÷ 365	
Mathematical behaviours	Marks
• identifies correct calculation	1

Question 3(e)

Solution	
6.00%	
Mathematical behaviours	Marks
• identifies effective annual interest = nominal with yearly compounding	1

Question 3(f)

Solution	
The greater the frequency of compounding the greater the effective interest rate	
Mathematical behaviours	Marks
• identifies relationship between variables	1

Question 3(g)

Solution	
6.173% The value must be less than 6.18% and more than 6.1678%	
Mathematical behaviours	Marks
• determines a suitable estimate • provides mathematical justification using data from the table.	1 1

TASK 16: PLANNING

Extended investigation

Unit 4

Topic 3: Project planning and scheduling using critical path analysis (CPA)

Course-related information

The concepts and skills covered in this investigation relate to the following content descriptors within the Australian Curriculum: General Mathematics course:

construct a network to represent the durations and interdependencies of activities that must be completed during the project (ACMGM104)

use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project (ACMGM105)

use ESTs and LSTs to locate the critical path(s) for the project (ACMGM106)

use the critical path to determine the minimum time for a project to be completed (ACMGM107)

calculate float times for non-critical activities (ACMGM108)

The ability to choose and use appropriate technology to enhance and extend concept development is also required for some of the items.

Background information

Students need to have had some exposure to project networks, but the preparation activities can serve to focus their efforts to prepare for this investigation.

Task conditions

It is suggested that the students are given the preparation activity approximately one week before the in-class validation. They should prepare solutions to the exercises notes on their work and bring these to the validation.

Planning

Extended investigation **Part 1: Preparation activity**

The preparation activity for this investigation consists of a series of exercises which should be completed before the validation component of the investigation. Students should bring two pages of notes to the validation and it is recommended that the notes include the answers to the exercises in the preparation activity and notes on dot points 4.3.4 to 4.3.8 from the course syllabus.

Exercise 1:

(a) Describe what is meant by a project in the context of this course.

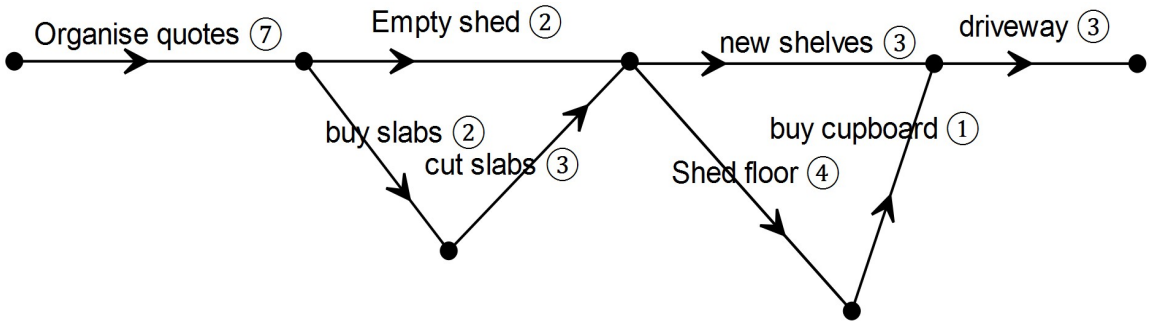
(b) Explain how a network diagram can be used to represent a project.

(c) Milly had to do a project on “Coral” for her Year 2 study of the environment. How could this assignment be an example of a project in the mathematical sense?

(d) Identify ten different examples of projects that could be studied in this course.

Exercise 2:

Study the network provided. Times are given in days.



(a) For each activity identify which other activities needed to be completed before it could start (immediate predecessors).

(b) How long did the renovation take altogether?

(c) Determine the float time for emptying the shed.

(d) In this network could the new shelves be built while the shed floor was being done?

(e) The rubbish generated from the cleaning and building activities had to be removed before the driveway started. If Tim started organising quotes on 1 September 2015, by what date did the rubbish need to be removed?

Exercise 3:

Construct a network to represent the tasks that needed to be completed in the following project. Show the order and interdependencies of the tasks and their duration.

Sarah applied for a position in an early learning centre. The tasks carried out were as follows with times and immediate predecessors provided.

Activity	Time (days)	Immediate predecessors
A: Sarah saw an advertisement for the position and investigated the documents needed for an application	2	none
B: Sarah waited for a Working With Children card which she had to take to the first interview	10	A
C: Sarah worked on her CV	2	A
D: Sarah's referees prepared her references	3	A
E: Sarah prepared her application form	3	C
F: Sarah decided to buy a new outfit after submitting the application	2	E, D
G: Sarah had to wait to be notified of her success in getting an interview	10	E, D
H: Sarah had two interviews	7	B,F,G
I: Sarah learned that she was successful	4	H

Use a logical process to determine the time that passed from Sarah seeing the advertisement until she learnt that she had the position.

Exercise 4:

Select one of the following "projects" and create your own network.

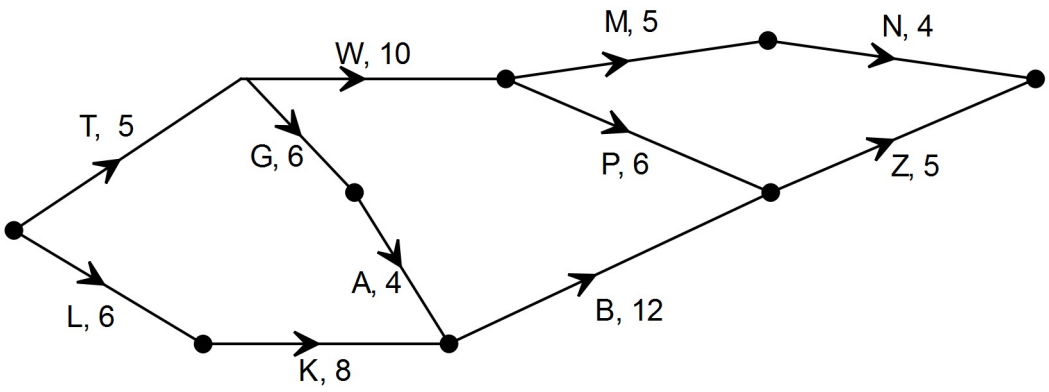
- Cooking your favourite meal
- Getting ready for work
- Getting together with friends
- Going to a sporting fixture
- Visiting an elderly relative

Exercise 5:

In this exercise the following terms are used:

forward scanning	backward scanning
earliest start times (EST)	latest start times (LST)
critical path	float times

The following project network represents some of the activities that need to be completed in preparing the early learning centre before the children arrive each morning. The estimated time for each activity is given in minutes.



(a) Determine the EST and LST for each activity.

(b) Use the ESTs and the LSTs to determine the critical path.

(c) Determine the minimum time needed for the centre to be prepared each morning.

(d) If the centre needs to be ready by 7:00 am each morning, when should the preparations commence?

(e) Identify the non-critical activities.

(f) If filling up the water bottles (W) needs more time, how much more time can be allocated without changing the overall time to prepare.

(g) Identify other activities that can take more time without changing the overall preparation time.

Question 2(d)

Solution	
RP, WL and AW	
Mathematical behaviours	Marks
• reduces the number of odd vertices, leaving only two odd vertices	1
• route goes through every vertex once only	1

Question 3(a)

Solution	
(i) W (ii) K (iii) 30 litres per minute (iv) 25 litres per minute	
Mathematical behaviours	Marks
• identifies source	1
• identifies sink	1
• determines maximum input to the system	1
• determines maximum output from the system	1

Question 3(b)

Solution	
Cannot exceed 1500 litres per hour	
Mathematical behaviours	Marks
• identifies maximum output at the sink cannot be exceeded	1

Question 3(c)

Solution	
6 along WQPK (QP=0) 4 along WQRK (WQ=0) 4 along WZPK (ZP=0) 4 along WZMK (MK=0) 1 along WTRK ((RK=0) 5 along WTMK (WT=0) Total 24 litres per minute	
Mathematical behaviours	Marks
• selects appropriate routes	1
• identifies maximum capacity for routes	1
• identifies all routes possible	1
• determines maximum flow	1

Question 3(d)

Solution	
It makes no difference because R still has a capacity of 6 litres per minute	
Mathematical behaviours	Marks
• concludes correctly	1
• justifies conclusion	1