Three Types of Tasks
- Extended Investigations
- In-class Investigations
- Investigative Questions

18 Assessment Tasks

Includes solutions with marking keys showing mathematical behaviours

Written by teachers for teachers

Authors: Joan Burfitt
Rom Cirillo
Margaret Denham

Edited by: Trudy Chick

Australian CURRICULUM

The Mathematical Association of Western Australia Inc.

FREE SAMPLE TASK

INVESTIGATIONS FOR TEACHING & LEARNING YEAR 12 MATHEMATICAL METHODS

Foreword

For each unit three types of tasks are included:

- **Extended teaching and learning investigation** – an investigation that includes a preparation activity which could be done in class, individually or in groups, in the student's own time **followed by** an in-class validation, so that students can reflect on their understanding
- Solutions are provided where practicable for the preparation activity/investigation. For the in-class validation, there are solutions and marking keys, which identify the mathematical behaviours that students may exhibit.
- Teachers are encouraged to share these with students, so they gain experience with WACE style marking keys.

- **In-class teaching and learning investigation** – an investigation for which no prior preparation is required.
- Solutions and marking keys are provided.
- This type of task will give students experience with investigative style questions that may be included as part of response type assessments.

- **Investigative questions** – a series of short questions, which test the student's ability to apply their learning, to justify their conclusions, to investigate and to generalise, or to solve problems.
- Such questions could be included in a response or examination assessment. Solutions and marking key are provided.

Screen shots have been produced using the CASIO ClassPad II emulator

Most graphs and diagrams have been produced using efofex software.

The syllabus dot points are reproduced here under a licence agreement with the SCHOOL CURRICULUM AND STANDARDS AUTHORITY. The Authority is acknowledged as the owner of all dot points from the respective syllabus.

© The investigations and solutions in this publication are for the sole use of staff and students of the purchasing school. It is a breach of copyright to distribute them in whole or part in any form now or in the future to any other school, company or person.

Students should not be given copies of the validation parts or the solutions to the investigations to take home.

 ©MAWA 2016

CONTENTS

MATHEMATICAL METHODS: Year 12

3.1 Further differentiation and applications

Task 1:	Extended investigation:	The second derivative	4
Task 2:	In-class investigation:	Taylor series	20
Task 3:	Investigative questions		31

3.2 Integrals

Task 4:	Extended investigation:	Area between functions	40
Task 5:	In-class investigation:	Areas between graphs and the x-axis	64
Task 6:	Investigative questions		78

3.3 Discrete random variables

Task7:	Extended investigation:	Distribution of sample means	93
Task 8:	In-class investigation:	Binomial distribution	111
Task 9:	Investigative questions		121

4.1 The logarithmic function

Task10:	Extended investigation:	Logarithms	134
Task 11:	In-class investigation:	The logarithm function	160
Task 12:	Investigative questions		175

4.2 Continuous random variables and the normal distribution

Task13:	Extended investigation:	The Normal distribution curve	188
Task 14:	In-class investigation:	Continuous distribution curves	219
Task 15:	Investigative questions		230

4.3 Interval estimates for proportions

Task16:	Extended investigation:	Simulating population distributions	241
Task 17:	In-class investigation:	Interval estimates	267
Task 18:	Investigative questions		281

©MAWA 2016

TASK 17

Confidence intervals for proportions

In-Class Investigation

Unit 4

Topic 4.3: Interval estimates of proportion

Course-related information

The concepts and skills included in this investigation relate to the following dot points within the ACARA Mathematical Methods syllabus:

Confidence intervals for proportions

- examine the concept of an interval estimate for a parameter associated with a random variable (ACMMM177)
- use the approximate confidence interval $\left(\hat{p} - z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}, \hat{p} + z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}\right)$ as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution (ACMMM178)
- define the approximate margin of error $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and understand the trade-off between margin of error and level of confidence (ACMMM179)
- use simulation to illustrate variations in confidence intervals between samples and to show that most, but not all, confidence intervals contain p (ACMMM180)

Background information

Students should have reviewed their knowledge of the content of the topic, *Continuous random variables and the normal distribution*. Students should have had at least an introduction to the concept of 'confidence intervals' for the population proportion and be aware of appropriate simulation routines to generate simulated sample proportion data for various distributions.

Task conditions

This task consists of an in-class investigation for which students may need 45 – 55 minutes to complete. Students do need to be familiar with the use of CAS technology to work with the normal distribution and to find confidence intervals for the population proportion. It is expected that they will have access to such technology during the investigation.

 ©MAWA 2016

Confidence intervals for proportions

In-class investigation **Total marks: 34**

Suppose that we want to know what proportion of the population of Perth have a smart phone?

As we do not know the value of the population proportion, p, that have a smart phone, we are interested in obtaining an estimate of p. A single observation x of X can be used to provide a *point estimate* of the unknown population proportion, $p = \frac{x}{n}$. The sample proportion $\hat{p}$ is an estimate of the population proportion, p.

We are also interested in quantifying the uncertainty of using $\hat{p}$ as an estimate of p. Although the underlying distribution would be binomial, as the sample size, n, increases, the distribution of the sample proportions will approach that of a normal distribution, with mean, p and standard deviation, $\sqrt{\frac{p(1-p)}{n}}$.

Question 1 **(17 marks)**

A survey of 500 Perth residents indicates that 340 had smart phones.

(a) Determine the sample proportion, $\hat{p}$, of the Perth residents surveyed that own a smart phone. (1)

Suppose that multiple simulations of the survey were to be performed. Each simulation of the survey will result in a different value for $\hat{p}$.

(b) State an estimate for the mean and standard deviation of the sample proportions, $\hat{p}$. (3)

Assuming that n is large enough, the distribution of the sample proportions, $\hat{p}$, may be considered to be normally distributed.

(c) If **ten** random samples of 500 Perth residents are simulated, in how many of the samples would you expect 350 or more residents to have a smart phone? Justify your estimate. (4)

 ©MAWA 2016

(d) To explain the estimate from part (c) to her superior, a researcher performed some simulations of the situation, the results of which are shown below.

```
randBin(500,0.68,10)/500⇒list1
{0.672,0.638,0.672,0.68,0.694,0.676,0.698,0.70
randBin(500,0.68,10)/500⇒list2
{0.672,0.68,0.678,0.652,0.702,0.678,0.662,0.71
randBin(500,0.68,10)/500⇒list3
{0.668,0.682,0.692,0.688,0.646,0.67,0.682,0.71
```

	list1	list2	list3	li
1	0.672	0.672	0.668	
2	0.638	0.68	0.682	
3	0.672	0.678	0.692	
4	0.68	0.652	0.688	
5	0.694	0.702	0.646	
6	0.676	0.678	0.67	
7	0.698	0.662	0.682	
8	0.702	0.71	0.712	
9	0.668	0.682	0.692	
10	0.654	0.652	0.708	
11				

Provide an interpretation of these results, indicating whether they support the result from part (c) or not. (5)

(e) A smart phone distribution company employed a researcher and requested that she use the survey information available to provide them with a **range** of the number of smart phones per 1000 Perth residents, of which they could be **confident**. Provide a sample answer that you suggest she might give. (4)

©MAWA 2016

Question 2 **(9 marks)**

The value of the sample proportion, $\hat{p}$, is going to change from sample to sample. Often, it is more appropriate to use an *interval estimate* which we can be reasonably be sure contains the population parameter, p .

Hence, we often use the approximate confidence interval $\left(\hat{p} - z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}, \hat{p} + z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}\right)$ as an interval estimate for p , where z is the appropriate quantile for the standard normal distribution.

In WA, a survey of attitudes towards the introduction of a refund for recycling drink containers (cans and glass and plastic bottles) was undertaken. A random sample of 1000 people were chosen, and each person was asked to answer the question:

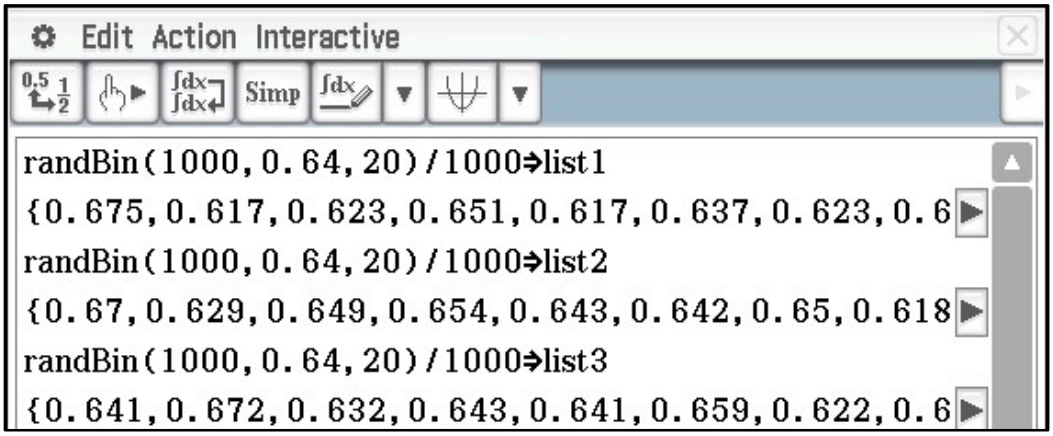
'Do you think the government should introduce a refund on returned drink containers (cans and bottles)?'

A total of 640 people answered 'yes' to the question.

(a) Based on the results of the survey, determine the 99% confidence interval for the proportion of the population of W.A. who agree with the introduction of the recycling refund. Provide an interpretative statement as to what the interval means. (3)

 ©MAWA 2016

A researcher conducted simulations of the survey to model likely responses. The results of the simulation are shown below.



	list1	list2	list3	list4
1	0.675	0.67	0.641	
2	0.617	0.629	0.672	
3	0.623	0.649	0.632	
4	0.651	0.654	0.643	
5	0.617	0.643	0.641	
6	0.637	0.642	0.659	
7	0.623	0.65	0.622	
8	0.644	0.618	0.654	
9	0.644	0.65	0.629	
10	0.654	0.646	0.662	
11	0.657	0.609	0.663	
12	0.634	0.644	0.62	
13	0.629	0.628	0.627	
14	0.654	0.682	0.628	
15	0.66	0.652	0.654	
16	0.637	0.654	0.629	
17	0.648	0.63	0.611	
18	0.653	0.635	0.66	
19	0.634	0.655	0.623	
20	0.632	0.631	0.628	
21				

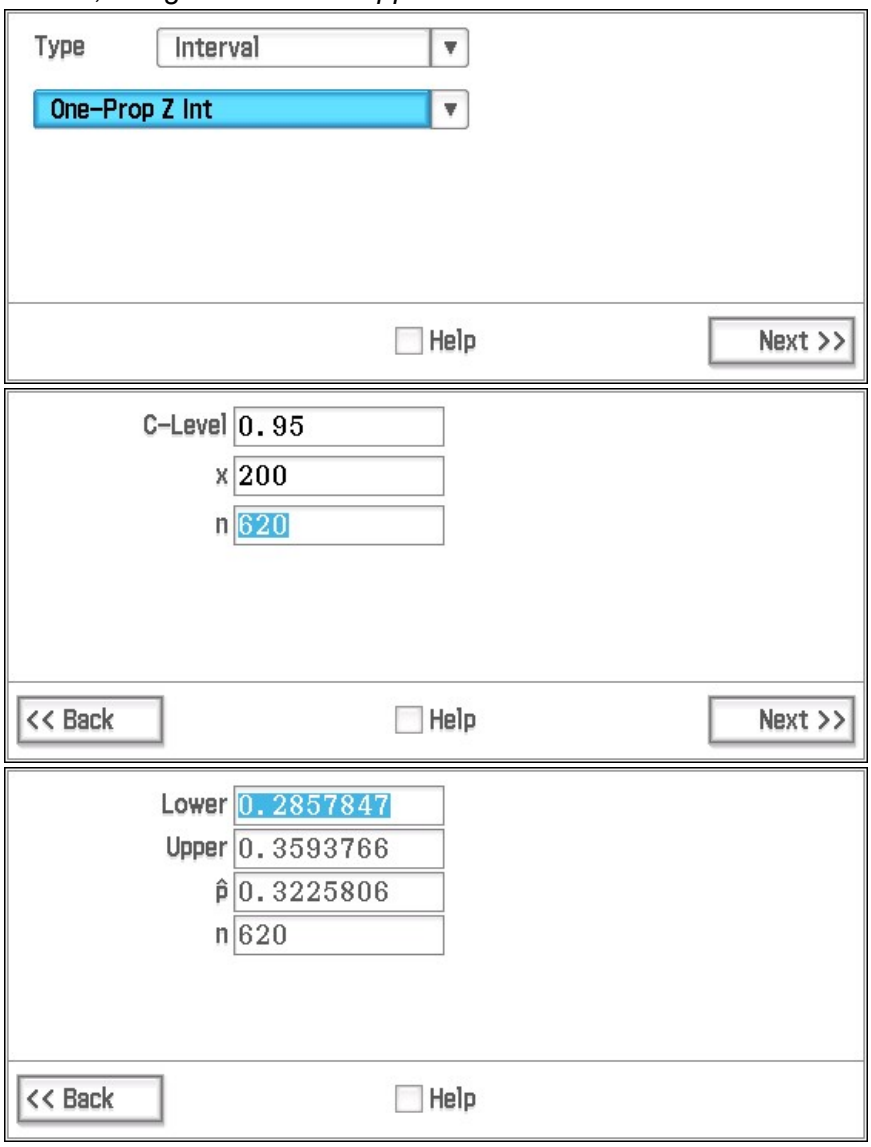
(b) Do any of the sample proportions generated by the simulation fall outside the confidence interval you found in part (a). Provide a possible interpretation of why this would be the case. (2)

©MAWA 2016

(c) Determine the 95% confidence interval for the proportion of the population of W.A. that agree with the introduction of the recycling refund. Do any of the sample proportions generated by the simulation fall outside the 95% confidence interval you found in part (b)? Provide a possible interpretation of why this would be the case. (4)

Question 3 **(8 marks)**

A random sample of 620 golfers found that 200 of them were left-handed.
The 95% confidence interval of the proportion of left-handed golfers may be found as follows, using the *statistics App* of a ClassPad calculator.



 ©MAWA 2016

Asked to provide an interpretation of this result, a company marketing employee states that based on the survey the proportion of golfers that are left-handed may be estimated to 32.26% and that the probability $P(0.2858 < p < 0.3594)$ =0.95.

(a) Write a brief comment, explaining whether the company employee has provided a valid and comprehensive interpretation. (4)

The margin of error (M) is described as the distance between a sample estimate and the endpoints of a confidence interval. M is given by the rule $M = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where z is the appropriate quantile for the standard normal distribution.

(b) Determine the margins of error for 95% and 90% confidence intervals for the proportion of left-handed golfers. (2)

(c) Explain briefly how the margin of error varies as the confidence interval changes. (2)

End of questions

©MAWA 2016

Confidence intervals for proportions

In-class investigation
Solutions and marking key

Question 1(a)

Solution	
$\hat{p} = \frac{340}{500} = 0.68$	
Mathematical behaviours	Marks
• correctly determines the sample proportion, $\hat{p}$	1

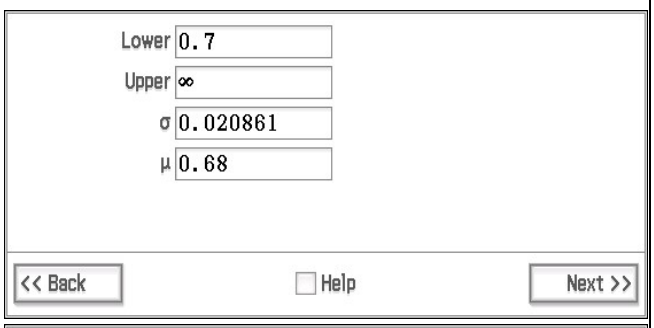
Question 1(b)

Solution	
The mean of $\hat{p}$ =0.68, and using the same $n = 500$, the standard deviation of $\hat{p} = \sqrt{\frac{0.68(1-0.68)}{500}} = 0.020861$	
Mathematical behaviours	Marks
• correctly states the mean of $\hat{p}$	1
• applies $\sqrt{\frac{p(1-p)}{n}}$ correctly	1
• states the correct standard deviation	1

©MAWA 2016

Question 1(c)

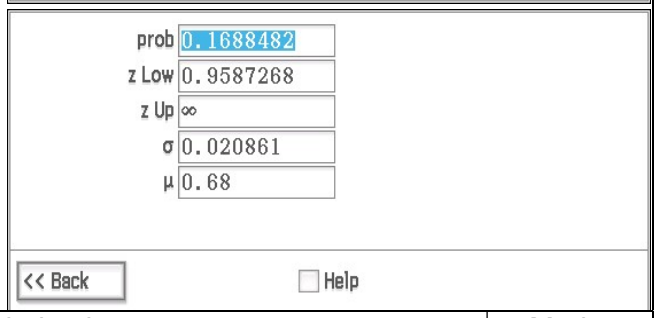
Solution
Assuming the sample proportions are normally distributed with a mean of 0.68 and standard deviation of 0.020861 (as found in part (b)) then we may calculate the probability that a randomly selected $\hat{p} > \frac{350}{500} = 0.7$ using the 'Normal CD' function on the calculator.



Hence, the probability that a particular $\hat{p}$ will be greater than 0.7 (i.e. that more than 350 out 500 residents will have a smart phone) is 0.1688.

Multiplying by 10, since there are 10 samples in the simulation, we get 1.688.

Of course, there will not be 1.688 samples with more than 350 residents having smart phones. We would expect 1 or 2 (most likely 2) samples to have 350 or more residents with smart phones.



Mathematical behaviours	Marks
• applies the normal distribution with correct mean and standard deviation	1
• calculates $P(\hat{p} > 0.7)$	1
• multiplies by 10 to get the number of samples	1
• interprets the meaning of the answer correctly	1

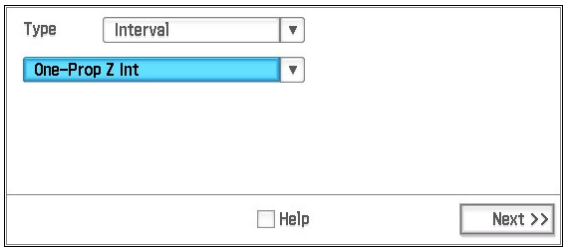
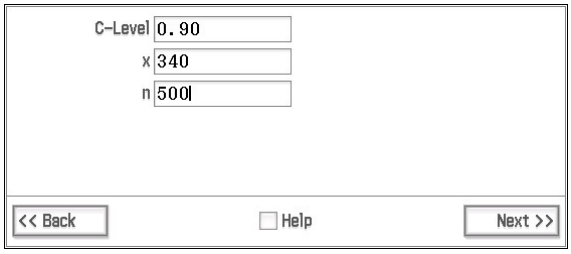
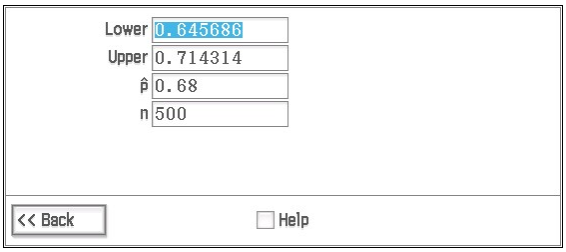
©MAWA 2016

Question 1(d)

Solution	
The information provided shows the simulation of 10 samples from a binomial distribution with a population proportion of 0.68 and a sample size of 500. The simulation is repeated 3 times and the values of the sample proportions, $\hat{p}$, are displayed in 3 separate lists. From list 1, there is only one sample proportion which is greater than 0.7 (i.e. there is only one of the 10 samples that has more than 350 residents with a smart phone). This is sample 8 with $\hat{p} = 0.702$. In list 2, there are two (samples 5 and 8) and in list 3 there are also two (samples 8 and 10). Hence these results support the theoretically calculated result from part (c) which also suggested that 1 or 2 of the 10 samples would have 350 or more Perth residents with a smart phone.	
Mathematical behaviours	Marks
• correctly identifies the information is from a binomial distribution	1
• identifies the numbers in the lists as sample proportions	1
• identifies the number of sample proportions > 0.7 in each list	1
• interprets the meaning of the $\hat{p}'s$ in terms of the context of the situation	1
• makes the appropriate connection with part (c).	1

©MAWA 2016

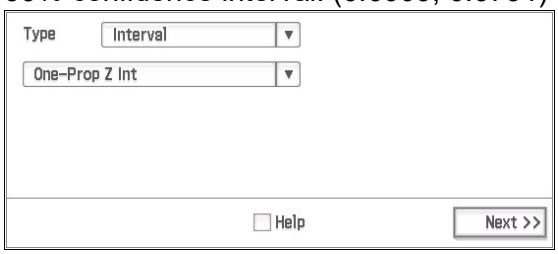
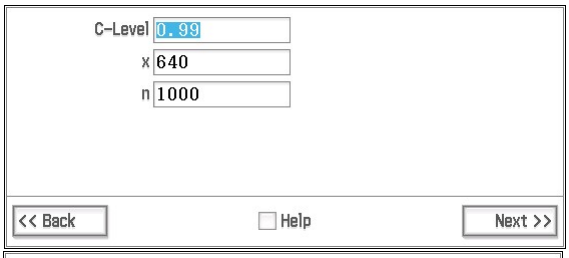
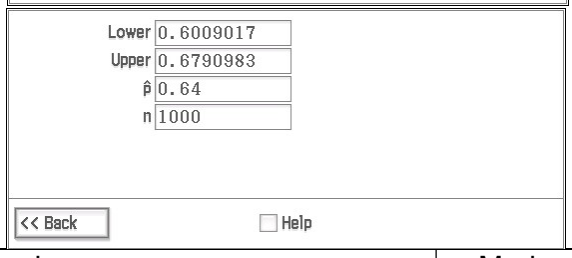
Question 1(e)

Solution
The researcher could calculate various confidence intervals and use them to provide the range requested to varying degrees of confidence. The 90% confidence interval for p could be calculated as follows:          From these results, the researcher could suggest that the number of residents with a smart phone would be between 645 and 714 (per 1000 Perth residents) and she could say this with 90% confidence. If the company wants more confidence, the researcher could suggest that between 639 and 721 (per 1000 Perth residents) have a smart phone, and she could say this with a 95% degree of confidence. The researcher could also point out that the higher the level of confidence that they require, the larger the interval range will be.

Mathematical behaviours	Marks
• correctly calculates the 90% confidence interval	1
• correctly calculates the 95% confidence interval	1
• provides suitable numbers by way of intervals	1
• correctly interprets the meaning of the numbers (in terms of confidence)	1

©MAWA 2016

Question 2(a)

Solution
Based on the survey, $p = 0.64$, $n = 1000$ 99% confidence interval: (0.6009, 0.6791)       Hence, we can be 99% confident that out of every 1000 people between 601 and 679 people agree with the recycling refund. 

Mathematical behaviours	Marks
• correctly determines the 99% confidence interval for p (either using the One-Prop z-interval on a CAS calculator or using $\left(\hat{p} - z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}, \hat{p} + z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}\right)$	2
• provides an interpretive statement as to the meaning of the interval, in the context of the situation	1

Question 2(b)

Solution
No, none of the $\hat{p}$ in the data lists provided fall outside the 99% confidence interval. This is because the 99% confidence interval is large enough to include, all the $\hat{p}$'s that are approximately, ± 3 standard deviations, either side of $\hat{p}$. This is most unlikely.

Mathematical behaviours	Marks
• correctly says, No.	1
• provides an acceptable interpretation	1

©MAWA 2016

Question 2(c)

Solution	
95% confidence interval: (0.6102, 0.6698) 4 out of the 60 sample proportions are outside the C.I., namely – below, 0.609 (list 2 number 11) and above, 0.675 (list 1 number 1), 0.682 (list 2 number 14), 0.672 (list 3 number 2). We would expect some of the $\hat{p}$'s to be outside the CI (it is not a 100% CI) but 4 out of 60 is actually on the high side for a 95% CI.	
Mathematical behaviours	Marks
• determine the 95% CI correctly • identifies 4 of $\hat{p}$'s as outside the CI • provide a suitable interpretative statement	2 1 1

Question 3(a)

Solution	
From the survey, the proportion of left handed golfers (p) may be estimated to be 0.3226 or 32.26% as provided in the statement by the marketing employee. However, it is not appropriate to say that $P(0.2858 < p < 0.3594) = 0.95$. This statement is .meaningless, as p either does or does not lie within the confidence interval. A better interpretation would be to have said that approximately 95% of such intervals will contain the population proportion, p. The CI will be different for each $\hat{p}$ as can be seen from the formula for the calculation of C.I.'s $\left(\hat{p} - z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}, \hat{p} + z\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}\right)$	
Mathematical behaviours	Marks
• indicates that the estimate for p, based on the survey results, is correct • identifies the probability statement as inappropriate • provides an appropriate statement for the interpretation of the confidence interval • indicates that there is a confidence interval dependent on each $\hat{p}$	1 1 1 1

Question 3(b)

Solution	
$M_{95} = 1.96\sqrt{\frac{0.3226(1-0.3226)}{620}} = 0.03680$ *changed to 0.03680 from 0.03697* $M_{90} = 1.65\sqrt{\frac{0.3226(1-0.3226)}{620}} = 0.03098$	
Mathematical behaviours	Marks
• calculates each margin of error correctly	1+1

©MAWA 2016

Question 3(c)

Solution	
As the degree of confidence for a confidence interval increases, from 90% to 95% for example, the margin of error decreases. This is because an increase in the degree of confidence increases the length of the confidence interval and so the chance of p being outside the confidence interval decreases.	
Mathematical behaviours	Marks
• provides a good, clear statement • provides any plausible statement (1 only)	2 (1)

©MAWA 2016